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Non-Markovian persistence at the parity conserving point of a one-dimensional nonequilibrium kinetic Ising model

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Abstract. One-dimensional nonequilibrium kinetic Ising models evolving under the competing effect of spin–flips at zero temperature and nearest-neighbour spin exchanges exhibiting a parity-conserving (PC) phase transition on the level of kinks are investigated here numerically from the point of view of the underlying spin system. The dynamical persistency exponent Θ and the exponent λ characterizing the two-time autocorrelation function of the total magnetization under nonequilibrium conditions are reported. It is found that the critical fluctuations at the PC transition have a strong effect on the spins: the behaviour becomes non-Markovian and the above exponents exhibit drastic changes as compared with the Markovian Glauber–Ising case. In this context the crucial importance of considering the global order parameter (instead of the local one) is emphasized.

In recent years, two nonequilibrium dynamical critical exponents have been discovered, which arise under nonequilibrium conditions. The nonequilibrium (short-time) exponent λ characterizes two-time correlations in systems relaxing to their critical state in the process of quenching from infinitely high temperatures to T_c [1, 2]. Recently, one more critical exponent was proposed [3], the persistence exponent Θ, associated with the probability $p(t) \propto t^{-\Theta}$, that the *global* order parameter has not changed sign up to time t after a quench to the critical point [4]. For some known examples, cited in [4], the scaling law

$$\Theta Z = \lambda - d + 1 - \frac{\eta}{2} \tag{1}$$

is satisfied (here d is the dimensionality and η is the static critical exponent of the order parameter correlation function), which has been derived assuming that the dynamics of the order parameter is a Markovian process. In general, however, λ and Θ have been proposed [4] to be independent, new critical dynamical exponents.

One of the soluble examples is the $d = 1$ Ising model with Glauber kinetics. In this case the critical temperature is at $T = 0$, the transition is of first order and as shown in [4], the persistence exponent is $\Theta = \frac{1}{4}$ for the global order parameter which is the total magnetization $M(t)$. Moreover, λ is known to be $\lambda = 1$ in this model. The aim of this paper is to study these new dynamical critical exponents in a simple *nonequilibrium* Ising system (NEKIM) introduced in [5]. In the plane of two characteristic parameters of the NEKIM transitions rates (δ and p_{ex}, see later) the phase diagram of NEKIM consists of a line of second-order phase transition points for the kinks from an absorbing to an active state which belongs to the parity conserving (PC) universality class [6, 7, 5, 8–10]. The

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absorbing phase is doubly degenerate, an initial state decays algebraically to the stationary state which is one of the absorbing ones (all spins up or all spins down, provided the initial state has an even number of kinks) and the whole absorbing phase behaves like a critical point with power-law decay of correlations, as at the Glauber–Ising point ($\delta = 0$, $p_{\rm ex} = 0$). This first-order transition at $T = 0$ of the Ising system disappears at the PC line and is, of course, absent in the whole active phase of the kinks.

The critical fluctuations of the PC transition exert a pronounced effect on the critical behaviour of the underlying spin system as found earlier [11] thus, for example the classical dynamical exponent Z, defined, as usual through $\tau \propto \xi^Z$ with $\xi \propto p^{-\nu}$ was found to be $Z = 1.75(1)$ instead of the Glauber–Ising value of $Z = 2$. (We note here that $p = {\rm e}^{-\frac{4J}{kT}}$ plays the role of $(T - T_{\rm c}/T_{\rm c})$ in one dimension with $T_{\rm c} = 0$ and the static exponents are defined as powers of p for $T \to 0$).

The question arises how the citical fluctuations of the PC transition affect the other two critical dynamical exponents Θ and λ.

Before entering into the details of our results for Θ and λ, the model will be described in some detail.

In NEKIM the system evolves under a combined effect of spin–flips and spin–exchanges. The spin–flip transition rate in one dimension for spin s_i ($s_i = \pm 1$) sitting at site i is [12]:

$$w_i = \frac{\Gamma}{2}(1 + \delta s_{i-1}s_{i+1})\left(1 - \frac{\gamma}{2}s_i(s_{i-1} + s_{i+1})\right) \tag{2}$$

where $\gamma = \tanh 2J/kT$ (J denoting the coupling constant in the Ising Hamiltonian), Γ and δ are further parameters. At $T = 0$, $\gamma = 1$ and there are two independent nonzero rates $\frac{\Gamma}{2}(1 - \delta)$ and $\frac{\Gamma}{2}(1 + \delta)$, responsible for random walk, and pairwise annihilation of kinks, respectively.

The spin–exchange transition rate of nearest-neighbour spins (the Kawasaki[13] rate at $T = \infty$) is $w_{ii+1} = \frac{1}{2}p_{\rm ex}[1 - s_i s_{i+1}]$, where $p_{\rm ex}$ is the probability of spin exchange. Spin–flip and spin–exchange have been applied alternately.

In this system, at $T = 0$, a PC-type phase transition takes place. In [5] we have started from a random initial state and determined the phase boundary in the $(\delta, p_{\rm ex})$ plane. In the following we will choose a typical (relatively free from transients) point on this phase diagram and make simulations at this point. The parameters chosen are: $\Gamma = 0.35$, $p_{\rm ex} = 0.3$, $\delta_{\rm c} = -0.395(2)$. In the simulations the spin–flip part has been applied using two-sublattice updating. We then stored the states of the spins and made L (size of the system) random attempts of exchange always using the stored situation for the states of the spins before updating. This all together is considered as one time-step of updating. (Usual MC update in this last step enhances the effect of $p_{\rm ex}$ and leads to $\delta_{\rm c} = -0.362(1)$.)

In [4] it has been argued that for studying nonequilibrium critical dynamics, the *global*, rather than the local order parameter should be considered. The nonequilibrium nature of the problem under consideration is due partly to the model itself and partly to the conditions of a quench from $T = \infty$ to 0. To realize such quench, we will restrict ourselves to completely random initial states and follow the behaviour of the system using the rules described above. The persistency exponent Θ is defined via the probability $p(t)$ that the *global* order parameter, which in our case is the total magnetization: $\langle M_{k=0}(t)\rangle = \frac{1}{L}\langle\sum_i s_i(t)\rangle$, has not changed sign up to time t:

$$p(t) \propto t^{-\Theta}. \tag{3}$$

Finite-size scaling (FSS) applied to the persistence problem [4] leads to the form,

$$p(t) = L^{-\Theta Z}g(t/L^Z). \tag{4}$$

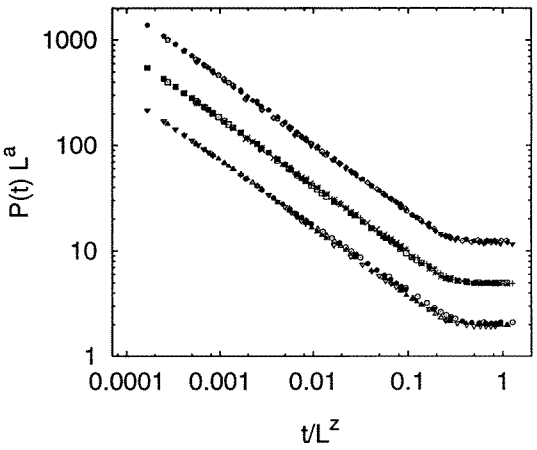

Figure 1. $p(t)L^a$, $a = \Theta Z$, plotted against t/L^Z with $Z = 1.75$ and $\Theta = 0.65, 0.67, 0.69$. For clarity the 0.65 and 0.69 data have been multiplied and divided by a factor of 2, respectively. The simulational data exhibited here have been obtained for $L = 100, 200, 400, 800$ sized systems labelled by different symbols, for statistical averages between 3×10^6–10^5 samples.

Simulations have been carried out in the range $50 \leqslant L \leqslant 2000$ with periodic, as well as antiperiodic, boundary conditions and for at least 10^5 independent runs. Figure 1 shows those of our results which have the best statistics with averages over up to 10^6 independent random initial configurations (those configurations, however, for which $M(0) = 0$ exactly, were discarded) with periodic boundary conditions for $L = 100, 200, 400$ and 800. Using $Z = 1.75(1)$ [11] the best fit corresponds to $\Theta = 0.67(1)$. Our simulations with antiperiodic boundary conditions, allowing only an odd number of kinks, have led to the same value of Θ as above though, of course, the form of the scaling function $g(t/L^Z)$ in this case is different. For comparison we have also simulated the exactly soluble Glauber–Ising case and found the expected value of $\Theta = 0.25$ within the accuracy of the simulations.

The local autocorrelation function defines the new exponent λ [1, 2]:

$$A(t, 0) = \frac{1}{L}\langle \sum_i s_i(0)s_i(t)\rangle \propto t^{-\frac{\lambda}{Z}}. \tag{5}$$

We have made simulations for this quantity, starting with a random initial configuration and allowing the system to evolve according to the rule of NEKIM as described above. Averages have been taken over random initial configurations in a chain of length $L = 1000$. Our results are shown in figure 2. The best fit has been obtained with $\lambda = 1.49(3)$, using $Z = 1.75(1)$. For comparison, numerical results for the corresponding quantity in the Glauber–Ising limiting case are also displayed in figure 2. It is worth mentioning, that data for $t \leqslant 10$ had to be discarded in both cases; power-law behaviour is seen only for later times; this does not change if the number of averages taken is increased even by an order of magnitude.

Following [4], we will now study the two-time autocorrelation function for the *global* order parameter: $A^{\text{global}}(t_1, t_2) = L\langle M_{k=0}(t_1)M_{k=0}(t_2)\rangle$ or rather its normalized form, namely

$$a(t_1, t_2) = A^{\text{global}}(t_1, t_2)/\sqrt{S(0, t_1)}\sqrt{S(0, t_2)} = f\left(\frac{t_1}{t_2}\right). \tag{6}$$

Here $S(0, t) = L\langle \frac{1}{L}[\sum_i s_i(t)]^2\rangle$ is the structure factor at the ferromagnetic peak and

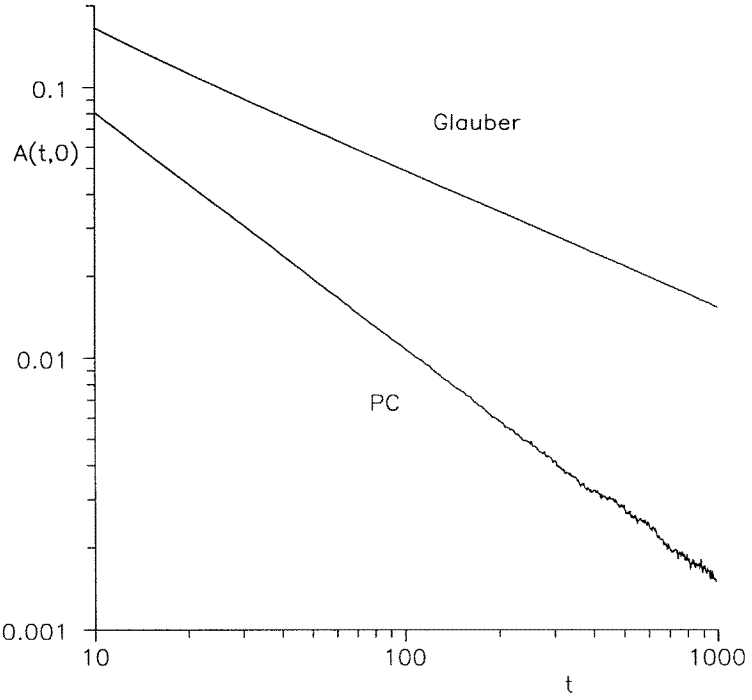

Figure 2. Time dependence of the local autocorrelation function $A(t, 0)$. $L = 1000$ and the number of averages over independent random initial states was 1.5×10^5. The Glauber–Ising case is shown for comparison.

the second equality follows from the scaling assumption. Moreover, for $y \to \infty$, $f(y) \sim y^{-(\lambda-d+1-\eta/2)/Z}$ is the expected power-law behaviour. Nevertheless, if the process is Markovian, the power-law behaviour of $f(t_1/t_2)$ has to hold for all $t_1 > t_2$ as shown in [4].

The second moment of the global magnetization (structure factor) should behave as [15]

$$S(0, t) \sim t^{\left(d-\frac{2\beta}{\nu}\right)/Z} \sim t^{(2-\eta)/Z}. \tag{7}$$

We have found earlier, in [11] that $\beta = 0.00(2)$, i.e. even at the PC point the Ising phase transition is of first order and thus $S(0, t) \propto t^{1/Z}$. Moreover, via the above applied scaling law $d - 2 + \eta = \frac{2\beta}{\nu}$, $\eta = 1.0(1)$ follows at the PC point, too.

Figure 3 shows $a(t_1/t_2)$ as a function of t_1/t_2 for five different values of t_2, $t_2 = 3, 10, 32, 50, 100$. We have simulated $A^{\text{global}}(t_1, t_2)$ while for the denominator we have used the power-law behaviour as indicated above with $1/Z = 0.57$. Unfortunately, it is very hard to get $A^{\text{global}}(t_1, t_2)$ to a satisfactory accuracy.

This is probably because, in the scaling form of $A^{\text{global}}(t_1, t_2)$, the leading-order term is $\propto (t_1/t_2)^{(d-\lambda)/Z}$ which is nonsingular in this model (it is marginal with $d = \lambda$ for the one-dimensional Glauber model while it is singular for the two-dimensional Ising case). This means that the $k = 0$ mode is not special for the two-time structure factor (but is for the equal-time structure factor); thus a power-law behaviour is a correction to scaling [18]. In more detail: the scaling form for the two-time structure factor can be written as

$$\langle M_{-k}(t_1) M_k(t_2)\rangle = t_2^{(2-\eta)/Z} \left[\frac{L(t_1)}{L(t_2)}\right]^{d-\lambda} f_1(kL(t_1))$$

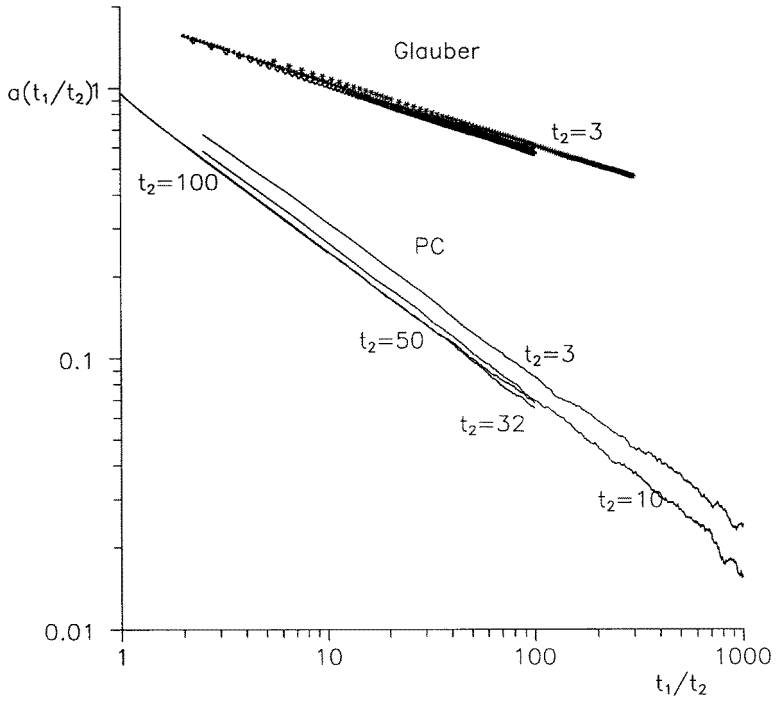

Figure 3. The normalized autocorrelation function $a(t_1/t_2)$ at the PC point for five different values of t_2 increasing in the downward direction in the range 3–100. The Galuber case is exhibited again for comparison. Here $t_2 = 3, 10, 32$, $L = 1000$ and the number of averages over independent initial states: $1–2 \times 10^5$.

$$+t_2{}^{(2-\eta)/Z}\left[\frac{L(t_1)}{L(t_2)}\right]^{\lambda_1} f_2(kL(t_1)) \qquad t_1, t_2 \gg 1 \qquad d > \lambda \tag{8}$$

where $L(t) \sim t^{1/Z}$ and for $k \to 0$ $f_1(kL(t)) \to$ constant. Moreover since the second term is a correction to scaling, $\lambda_1 < d - \lambda$. For $t_1 = t_2$ we get the usual structure factor and for $k \to 0$, without the correction term, this is the form cited in [4] below their equation (16). In this case, as we know the singular term is missing, we can say $\lim_{k\to 0} f_1(kL(t)) = 0$, scaling can still be present and λ_1 plays the role of $d - \lambda$ (we will use $d - \lambda$ for λ_1 in the following even for $\lambda > d$).

Now, returning to figure 3, apart from the first three decades in time, fluctuations prevent us from drawing any conclusions concerning the (correction to scaling) behaviour of $A^{\text{global}}(t_1, t_2)$, even for averages of order 10^5. For the quantity t_1/t_2 this fact narrows down the interval of analysable data even more. Nevertheless, it is clearly seen that the dynamic-scaling assumption expressed in equation (6) (which eventually can be expected to hold only for $t_1 \gg 1, t_2 \gg 1$!) is fulfilled to an accuracy better than 1% only for values $t_1 > t_2 \gtrsim 50$. In fact this is not typical: it has been proposed [14] that in the case of systems quenched to their critical temperature (here $T_c = 0$) universality and scaling may appear at quite an early stage of time evolution, far from equilibrium, where $\xi(t) \sim t^{1/Z}$ is still small. Based on the scaling relation for such early time intervals, a new way for measuring static and dynamic exponents has been proposed [15, 16] and also applied for the local autocorrelation function [17]. Indeed, some of our earlier results also show that power-law behaviour sets in for quite early times. Thus, for example in [11], concerning

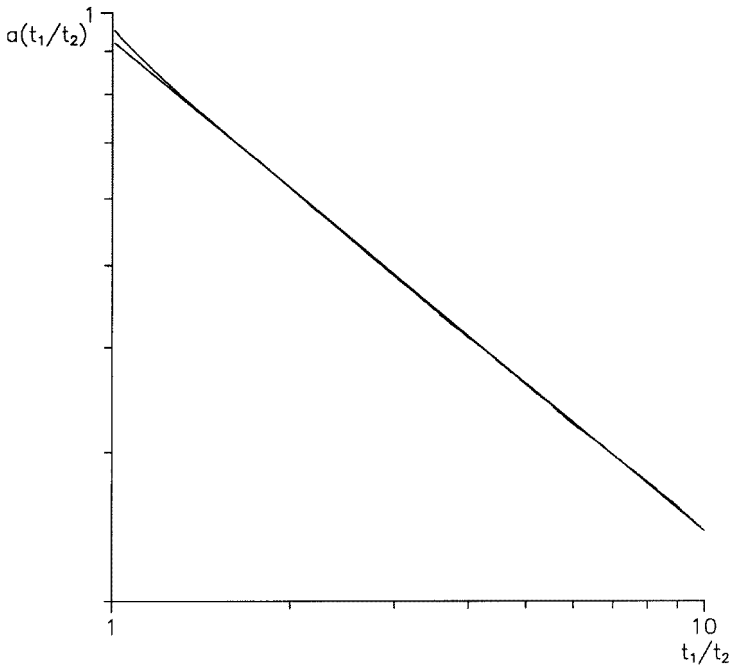

Figure 4. The dependence of the normalized autocorrelation function on t_1/t_2 in its first decade for $t_2 = 100$. $L = 1000$ and the number of averages was 10^6. The straight line is a best power-law fit with $(\lambda - \frac{\eta}{2})/Z = 0.581$.

the structure factor $S(0,t) \sim t^x$, the power-law behaviour was already apparent for very early times and for values of L as low as $L = 128$, provided the number of averages in the simulation was high enough (above 10^5). The obtained result, $x = 0.57 = 1/Z$ was used above.

For comparison we have carried out similar simulations of $A^{\text{global}}(t_1, t_2)$ for the exactly soluble Ising–Glauber case ($p_{\text{ex}} = 0, \delta = 0$), some of these are also exhibited in figure 3. Here dynamic scaling is fulfilled (to a similar accuracy as above) already for $t_1 > t_2 \gtrsim 5$ and the expected power-law behaviour is seen within error. It is worth mentioning that a similar value for λ, i.e. $\lambda = 1.0$ results from simulations in the whole absorbing region (thus e.g. for $p_{\text{ex}} = 0.35, \delta = 0$).

In order to establish whether the process is Markovian or not at the PC point it will be sufficient to examine the first decade in the variable t_1/t_2 in a region where dynamic scaling holds. Figure 4 shows the result for the case $t_2 = 100$, for averages over 10^6 independent initial states, again taking $L = 1000$. For the exponent $(\lambda - \eta/2)/Z$ the value 0.58(1) results as a best fit, giving $\lambda = 1.51(1)$ which is in accord, within error, with the value obtained above from the local autocorrelation function. Moreover, according to equation (1), the same exponent should equal Θ. Thus, supposing the Markovian property to hold has led to contradiction because the measured value of Θ is 0.67(1).

These results together with critical exponents obtained earlier in [11] are summarized in table 1.

In summary, we have carried out numerical simulations to investigate the nonequilibrium dynamic critical exponents Θ and λ with the aim to check the Markovian nature of the nonequilibrium Ising system in one dimension at the parity conserving phase transition point

Table 1. Simulation data for static and dynamic critical exponents for NEKIM.

	β	γ	ν	Z	Θ	λ
Glauber–Ising	0	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{1}{4}$	1
PC	0.00(1)	0.444(2)	0.444(2)	1.75(1)	0.67(1)	1.50(2)

of the phase diagram of NEKIM. On the basis of of these results we have been led to the conclusion that the effect of fluctuations felt by the spin system at the PC transition is such that the dynamical process becomes non-Markovian. The difference is quite pronounced, definitely beyond numerical errors. It should be emphasized, however, that the elementary NEKIM process is, of course, Markovian and in arriving at the above conclusion the fact that the relaxation of the *global* order parameter has been considered is of paramount importance.

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